APPENDIX B

SOME USEFUL TABLES

This Appendix gives some tables which tend to be useful in the analysis of repeat-buying behaviour along the lines discussed in this book.

Table B1 relates to the LSD and gives the value of the LSD parameter q for a given value of w. The range of values of w is from 1 .O to 10.9 and is therefore much more detailed than the brief extract in Table 2.2 of Chapter 2. However, interpolation in this table (let alone extrapolation for w >10) is relatively difficult because many of the values of q are close to 1. A better approach is to estimate the LSD parameter $a = q/(1+q)$ from w (and then either work with a or calculate q as $a/(1-a)$). A table for reading off a for a given value of w is given in Table B3.

Table B2 essentially gives Naperian or Natural logarithms for numbers running from 0 to 2. This is useful for computing expressions such as $\ln(1-q)$ and $\ln(1+q)$ which appear in the LSD theory.

For convenience, the tables are given in two parts. Table B2a gives $\ln(1-q)$ directly for values of q from .OOO to .999 (the values of $\ln(1-q)$ are always negative). This table can also be used to read off $\ln(1-b)$ for a given b, as is required for Table B3. Table B2b gives $\ln(1+q)$ for the same range of q.

Table B3 provides a way of reading off the parameter a in either the NBD or the LSD theory. (The two values will differ for any given data, although only slightly if the parameters are such that the LSD is a good approximation to the NBD.)

For the NBD, we have to calculate first the value of $c = -m/\ln(1-b)$ from the observed values of the mean m and the proportion of buyers b (Table B2a can be used to read off $\ln(1-b)$ for the given value of b). We then read off the value of the NBD parameter a for the calculated value of c from the Table B3. NBD parameter k can be calculated from $k = m/a$.

For the LSD, we can read the LSD values of a directly by entering the table with the observed value of w (where w = m/b).

Tables B4–B 7 give the NBD and LSD values of four basic repeat-buying statistics in two equal periods under stationary conditions, as follows:

Table B4: $100b_R/b$, i.e. the percentage of the buyers (b) in one time-period who buy the item again in another equal period.

Table B5: w_R, the average purchase frequency per repeat-buyer in one of the periods.

Table B6: $100b_N/b$, i.e. the percentage of buyers in one period who do NOT buy the item in another equal period (where the proportion of "lapsed" buyers is $b_L = b_N = 1 - b_R$).

Table B7: w_N, i.e. the average purchase frequency per "new" (or per "lapsed") buyer in the period in which they buy at all ($w_N = w_L$).

Values are given for the NBD and LSD and "approximation" formulae which were set out in Chapters 4, 7 and 8. In all cases, the observed values of b and m (for the NBD) or of w (for the LSD) in *one* of the two time-periods are used to enter the tables.

Only a limited range of values of b and w is covered. The tables are mainly meant to give a feel of how repeat-buying varies numerically with b and w, showing also how for low values of b the NBD values all closely approximate the LSD ones. (For detailed numerical work, direct calculation of the various statistics required rather than reading values off from extensive tables tends to be more useful.)

Tables B8–B9 illustrate the variation with the length of analysis-period Of the penetration b_T and of the average purchase frequency w_T in a time-period of length T (relative to some chosen period of arbitrary "unit" length). This has been done by showing b_T and w_T in each case as a fraction of the value of b_1 and w_1 in the unit time-period. To obtain values of $100b_T$ and w_T simply multiply by the values of $100b_1$ and w_1 given in the centre columns of these tables. When interpolating in these tables work in terms of b_T/b_1 and w_T/w_1, and only as a final stage multiply up by the observed values of $100b_1$ and w_1 to obtain $100b_T$ and w_T.

The last block in each table results from the LSD formulation of the model. This is the limiting case as b tends to zero, and the two blocks have been labelled $100b_1 = 0$ accordingly. They may be used either to obtain LSD values for b_T/b_1 and w_T/w_1 (and hence b_T and w_T) when b_1 is small, or to interpolate for the NBD model in cases where the value of $100b_1$ is less than 5%.

Note that NBD's do not exist for high values of b_1 when w_1 is low.

B fo from

w	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1.0	.000	.171	.298	.396	.472	.534	.584	.625	.660	.690
2.0	.715	.737	.757	.774	.789	.802	.814	.825	.834	.843
3.0	.851	.858	.865	.871	.877	.882	.887	.892	.896	.900
4.0	.903	.907	.910	.913	.916	.919	.921	.924	.926	.928
5.0	.930	.932	.934	.936	.937	.939	.940	.942	.943	.945
6.0	.946	.947	.948	.950	.951	.952	.953	.954	.955	.956
7.0	.956	.957	.958	.959	.960	.960	.961	.962	.962	.963
8.0	.964	.964	.965	.965	.966	.967	.967	.968	.968	.969
9.0	.969	.969	.970	.970	.971	.971	.972	.972	.972	.973
10.0	.973	.973	.974	.974	.974	.975	.975	.975	.976	.976

Table B.2a. Values of $\ln(1-q)$ for Values of q from .000 to .999

(ln stands for the Naperian or Natural Logarithm)

q	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.00	−.000	−.001	−.002	−.003	−.004	−.005	−.006	−.007	−.008	−.009
.01	−.010	−.011	−.012	−.013	−.014	−.015	−.016	−.017	−.018	−.019
.02	−.020	−.021	−.022	−.023	−.024	−.025	−.026	−.027	−.028	−.029
.03	−.030	−.031	−.033	−.034	−.035	−.036	−.037	−.038	−.039	−.040
.04	−.041	−.042	−.043	−.044	−.045	−.046	−.047	−.048	−.049	−.050
.05	−.051	−.052	−.053	−.054	−.056	−.057	−.058	−.059	−.060	−.061
.06	−.062	−.063	−.064	−.065	−.066	−.067	−.068	−.069	−.070	−.071
.07	−.073	−.074	−.075	−.076	−.077	−.078	−.079	−.080	−.081	−.082
.08	−.083	−.084	−.086	−.087	−.088	−.089	−.090	−.091	−.092	−.093
.09	−.094	−.095	−.097	−.098	−.099	−.100	−.101	−.102	−.103	−.104
.10	−.105	−.106	−.108	−.109	−.110	−.111	−.112	−.113	−.114	−.115
.11	−.117	−.118	−.119	−.120	−.121	−.122	−.123	−.124	−.126	−.127
.12	−.128	−.129	−.130	−.131	−.132	−.134	−.135	−.136	−.137	−.138
.13	−.139	−.140	−.142	−.143	−.144	−.145	−.146	−.147	−.149	−.150
.14	−.151	−.152	−.153	−.154	−.155	−.157	−.158	−.159	−.160	−.161
.15	−.163	−.164	−.165	−.166	−.167	−.168	−.170	−.171	−.172	−.173
.16	−.174	−.176	−.177	−.178	−.179	−.180	−.182	−.183	−.184	−.185
.17	−.186	−.188	−.189	−.190	−.191	−.192	−.194	−.195	−.196	−.197
.18	−.198	−.200	−.201	−.202	−.203	−.205	−.206	−.207	−.208	−.209
.19	−.211	−.212	−.213	−.214	−.216	−.217	−.218	−.219	−.221	−.222
.20	−.223	−.224	−.226	−.227	−.228	−.229	−.231	−.232	−.233	−.234
.21	−.236	−.237	−.238	−.240	−.241	−.242	−.243	−.245	−.246	−.247
.22	−.248	−.250	−.251	−.252	−.254	−.255	−.256	−.257	−.259	−.260
.23	−2.61	−.263	−.264	−.265	−.267	−.268	−.269	−.271	−.272	−.273
.24	−.274	−.276	−.277	−.278	−.280	−.281	−.282	−.284	−.285	−.286

Table B2a continued: ln(1−q)

q	.000	.001	.002	.003	.004	.005	.006	007	.008	.009
.25	−.288	−.289	−.290	−.292	−.293	−.294	−.296	−.297	−.298	−.300
.26	−.301	−.302	−.304	−.305	−.307	−.308	−.309	−.311	−.312	−.313
.27	−.315	−.316	−.317	−.319	−.320	−.322	−.323	−.324	−.326	−.327
.28	−.329	−.330	−.331	−.333	−.334	−.335	−.337	−.338	−.340	−.341
.29	−.343	−.344	−.345	−.347	−.348	−.350	−.351	−.352	−.354	−.355
.30	−.357	−.358	−.360	−.361	−.362	−.364	−.365	−.367	−.368	−.370
.31	−.371	−.373	−.374	−.375	−.377	−.378	−.380	−.381	−.383	−.384
.32	−.386	−.387	−.389	−.390	−.392	−.393	−.395	−.396	−.398	−.399
.33	−.401	−.402	−.403	−.405	−.406	−.408	−.409	−.411	−.413	−.414
.34	−.416	−.417	−.419	−.420	−.422	−.423	−.425	−.426	−.428	−.429
.35	−.431	−.432	−.434	−.435	−.437	−.439	−.440	−.442	−.443	−.445
.36	−.446	−.448	−.449	−.451	−.453	−.454	−.456	−.457	−.459	−.460
.37	−.462	−.464	−.465	−.467	−.468	−.470	−.472	−.473	−.475	−.476
.38	−.478	−.480	−.481	−.483	−.485	−.486	−.488	−.489	−.491	−.493
.39	−.494	−.496	−.498	−.499	−.501	−.503	−.504	−.506	−.508	−.509
.40	−.511	−.513	−.514	−.516	−.518	−.519	−.521	−.523	−.524	−.526
.41	−.528	−.529	−.531	−.533	−.534	−.536	−.538	−.540	−.541	−.543
.42	−.545	−.546	−.548	−.550	−.552	−.553	−.555	−.557	−.559	−.560
.43	−.562	−.564	−.566	−.567	−.569	−.571	−.573	−.575	−.576	−.578
.44	−.580	−.582	−.583	−.585	−.587	−.589	−.591	−.592	−.594	−.596
.45	−.598	−.600	−.602	−.603	−.605	−.607	−.609	−.611	−.613	−.614
.46	−.616	−.618	−.620	−.622	−.624	−.626	−.627	−.629	−.631	−.633
.47	−.635	−.637	−.639	−.641	−.642	−.644	−.646	−.648	−.650	−.652
.48	−.654	−.656	−.658	−.660	−.662	−.664	−.666	−.668	−.669	−.671
.49	−.673	−.675	−.677	−.679	−.681	−.683	−.685	−.687	−.689	−.691

Table B2a continued: $\ln(1-q)$

q	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.50	− .693	− .695	− .697	− .699	− .701	− .703	− .705	− .707	− .709	− .711
.51	− .713	− .715	− .717	− .720	− .722	− .724	− .726	− .728	− .730	− .732
.52	− .734	− .736	− .738	− .740	− .742	− .744	− .747	− .749	− .751	− .753
.53	− .755	− .757	− .759	− .761	− .764	− .766	− .768	− .770	− .772	− .774
.54	− .777	− .779	− .781	− .783	− .785	− .788	− .790	− .792	− .794	− .796
.55	− .799	− .801	− .803	− .805	− .807	− .810	− .812	− .814	− .817	− .819
.56	− .821	− .823	− .826	− .828	− .830	− .832	− .835	− .837	− .839	− .842
.57	− .844	− .846	− .849	− .851	− .853	− .856	− .858	− .860	− .863	− .865
.58	− .868	− .870	− .872	− .875	− .877	− .880	− .882	− .884	− .887	− .889
.59	− .892	− .894	− .897	− .899	− .901	− .904	− .906	− .909	− .911	− .914
.60	− .916	− .919	− .921	− .924	− .926	− .929	− .931	− .934	− .937	− .940
.61	− .942	− .944	− .947	− .949	− .952	− .955	− .957	− .960	− .962	− .965
.62	− .968	− .970	− .973	− .976	− .978	− .981	− .984	− .986	− .989	− .992
.63	− .994	− .997	-1 .000	-1.002	-1.005	-1.008	-1.011	-1.013	-1.016	-1.019
.64	-1.022	-1.025	-1.027	-1.030	-1.033	-1.036	-1.039	-1.041	-1.044	-1.047
.65	-1.050	-1.053	-1.056	-1.059	-1.061	-1.064	-1.067	-1.070	-1.073	-1.076
.66	-1.079	-1.082	-1.085	-1.088	-1.091	-1.094	-1.097	-1.100	-1.103	-1.106
.67	-1.109	-1.112	-1.115	-1.118	-1.121	-1.124	-1.127	-1.130	-1.133	-1.136
.68	-1.140	-1.143	-1.146	-1.149	-1.152	-1.155	-1.158	-1.162	-1.165	-1.168
.69	-1.171	-1.175	-1.178	-1.181	-1.184	-1.188	-1.191	-1.194	-1.197	-1.201
.70	-1.204	-1.207	-1.211	-1.214	-1.218	-1.221	-1.224	-1.228	-1.231	-1.235
.71	-1.238	-1.241	-1.245	-1.248	-1.252	-1.255	-1.259	-1.262	-1.266	-1.270
.72	-1.273	-1.277	-1.280	-1.284	-1.287	-1.291	-1.295	-1.298	-1.302	-1.306
.73	-1.309	-1.313	-1.317	-1.321	-1.324	-1.328	-1.332	-1.336	-1.340	-1.343
.74	-1.347	-1.351	-1.355	-1.359	-1.363	-1.367	-1.371	-1.375	-1.378	-1.382

Table **B2a** continued: ln(l-q)

q	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.75	-1.386	-1.390	-1.394	-1.399	-1.403	-1.407	-1.411	-1.415	-1.419	-1.423
.76	-1.427	-1.431	-1.436	-1.440	-1.444	-1.448	-1.453	-1.457	-1.461	-1.465
.77	-1.470	-1.474	-1.479	-1.483	-1.487	-1.492	-1.496	-1.501	-1.505	-1.510
.78	-1.514	-1.519	-1.523	-1.528	-1.533	-1.537	-1.542	-1.547	-1.551	-1.556
.79	-1.561	-1.566	-1.570	-1.575	-1.580	-1.585	-1.590	-1.595	-1.600	-1.605
.80	-1.610	-1.615	-1.620	-1.625	-1.630	-1.635	-1.640	-1.645	-1.650	-1.656
.81	-1.661	-1.666	-1.672	-1.677	-1.682	-1.688	-1.693	-1.698	-1.704	-1.709
.82	-1.715	-1.721	-1.726	-1.732	-1.737	-1.743	-1.749	-1.755	-1.760	-1.766
.83	-1.772	-1.778	-1.784	-1.790	-1.796	-1.802	-1.808	-1.814	-1.820	-1.827
.84	-1.833	-1.839	-1.845	-1.852	-1.858	-1.865	-1.871	-1.878	-1.884	-1.891
.85	-1.897	-1.904	-1.911	-1.918	-1.924	-1.931	-1.938	-1.945	-1.952	-1.959
.86	-1.966	-1.974	-1.981	-1.988	-1.995	-2.003	-2.010	-2.018	-2.025	-2.033
.87	-2.041	-2.048	-2.056	-2.064	-2.072	-2.080	-2.088	-2.096	-2.104	-2.112
.88	-2.121	-2.129	-2.137	-2.146	-2.155	-2.163	-2.172	-2.181	-2.190	-2.199
.89	-2.208	-2.217	-2.226	-2.235	-2.245	-2.254	-2.264	-2.273	-2.283	-2.293
.90	-2.303	-2.313	-2.323	-2.333	-2.344	-2.354	-2.365	-2.376	-2.386	-2,397
.91	-2.408	-2.420	-2.431	-2.442	-2.454	-2.466	-2.477	-2.489	-2.502	-2.5 14
.92	-2.526	-2.539	-2.552	-2.565	-2.578	-2.591	-2.604	-2.618	-2.632	-2.646
.93	-2.660	-2.674	-2.689	-2.704	-2.7 19	-2.734	-2.750	-2.765	-2.781	-2.798
.94	-2.814	-2.831	-2.848	-2.865	-2.883	-2.901	-2.920	-2.938	-2.957	-2.977
.95	-2.997	-3,017	-3.037	-3.059	-3.080	-3.102	-3.125	-3.148	-3.171	-3.195
.96	-3.220	-3.245	-3.271	-3.298	-3.325	-3.354	-3.383	-3.413	-3.443	-3.475
.97	-3.508	-3.540	-3.577	-3.614	-3.651	-3.691	-3.732	-3.774	-3.819	-3.865
.98	-3.912	-3.963	-4.017	-4.075	-4.135	-4.200	-4.269	-4.343	-4.423	-4.510
.99	-4.605	-4.711	-4.828	-4.962	-5.116	-5.298	-5.521	-5.809	-6.215	-6.908

Table B2b. Values of $\ln(1+q)$ for Values of q from .000 to .999

q	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.00	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.01	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019
.02	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
.03	.030	.031	.031	.032	.033	.034	.035	.036	.037	.038
.04	.039	.040	.041	.042	.043	.044	.045	.046	.047	.048
.05	.049	.050	.051	.052	.053	.054	.054	.055	.056	.057
.06	.058	.059	.060	.061	.062	.063	.064	.065	.066	.067
.07	.068	.069	.069	.070	.071	.072	.073	.074	.075	.076
.08	.077	.078	.079	.080	.081	.082	.082	.083	.084	.085
.09	.086	.087	.088	.089	.090	.091	.092	.093	.093	.094
.10	.095	.096	.097	.098	.099	.100	.101	.102	.103	.103
.11	.104	.105	.106	.107	.108	.109	.110	.111	.111	.112
.12	.113	.114	.115	.116	.117	.118	.119	.119	.120	.121
.13	.122	.123	.124	.125	.126	.127	.127	.128	.129	.130
.14	.131	.132	.133	.134	.134	.135	.136	.137	.138	.139
.15	.140	.141	.141	.142	.143	.144	.145	.146	.147	.147
.16	.148	.149	.150	.151	.152	.153	.154	.154	.155	.156
.17	.157	.158	.159	.159	.160	.161	.162	.163	.164	.165
.18	.165	.166	.167	.168	.169	.170	.171	.171	.172	.173
.19	.174	.175	.176	.176	.177	.178	.179	.180	.181	.181
.20	.182	.183	.184	.185	.186	.186	.187	.188	.189	.190
.21	.191	.191	.192	.193	.194	.195	.195	.196	.197	.198
.22	.199	.200	.200	.201	.202	.203	.204	.204	.205	.206
.23	.207	.208	.209	.209	.210	.211	.212	.213	.213	.214
.24	.215	.216	.217	.217	.218	.219	.220	.221	.221	.222

Table B2b continued: ln(1+q)

q	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.25	.223	.224	.225	.225	.226	.227	.228	.229	.229	.230
.26	.231	.232	.233	.233	.234	.235	.236	.237	.237	.238
.27	.239	.240	.240	.241	.242	.243	.244	.244	.245	.246
.28	.247	.248	.248	.249	.250	.251	.251	.252	.253	.254
.29	.255	.255	.256	.257	.258	.258	.259	.260	.261	.261
.30	.262	.263	.264	.265	.265	.266	.267	.268	.268	.269
.31	.270	.271	.271	.272	.273	.274	.274	.275	.276	.277
.32	.277	.278	.279	.280	.281	.281	.282	.283	.284	.284
.33	.285	.286	.287	.287	.288	.289	.290	.290	.291	.292
.34	.293	.293	.294	.295	.296	.296	.297	.298	.298	.299
.35	.300	.301	.301	.302	.303	.304	.304	.305	.306	.307
.36	.307	.308	.309	.310	.310	.311	.312	.312	.313	.314
.37	.315	.315	.316	.317	.318	.318	.319	.320	.320	.321
.38	.322	.323	.323	.324	.325	.326	.326	.327	.328	.328
.39	.329	.330	.331	.331	.332	.333	.333	.334	.335	.336
.40	.336	.337	.338	.338	.339	.340	.341	.341	.342	.343
.41	.343	.344	.345	.346	.346	.347	.348	.348	.349	.350
.42	.350	.351	.352	.353	.353	.354	.355	.355	.356	.357
.43	.358	.358	.359	.360	.360	.361	.362	.362	.363	.364
.44	.364	.365	.366	.367	.367	.368	.369	.369	.370	.371
.45	.371	.372	.373	.373	.374	.375	.376	.376	.377	.378
.46	.378	.379	.380	.380	.381	.382	.382	.383	.384	.384
.47	.385	.386	.386	.387	.388	.388	.389	.390	.391	.391
.48	.392	.393	.393	.394	.395	.395	.396	.397	.397	.398
.49	.399	.399	.400	.401	.401	.402	.403	.403	.404	.405

Table B2b continued: $\ln(1+q)$

q	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.50	.405	.406	.407	.407	.408	.409	.409	.410	.411	.411
.51	.412	.413	.413	.414	.415	.415	.416	.417	.417	.418
.52	.419	.419	.420	.420	.421	.422	.422	.423	.424	.424
.53	.425	.426	.426	.427	.428	.428	.429	.430	.430	.431
.54	.432	.432	.433	.434	.434	.435	.435	.436	.437	.437
.55	.438	.439	.439	.440	.441	.441	.442	.443	.443	.444
.56	.444	.445	.446	.446	.447	.448	.448	.449	.450	.450
.57	.451	.452	.452	.453	.453	.454	.455	.455	.456	.457
.58	.457	.458	.458	.459	.460	.460	.461	.462	.462	.463
.59	.464	.464	.465	.465	.466	.467	.467	.468	.469	.469
.60	.470	.470	.471	.472	.472	.473	.474	.474	.475	.475
.61	.476	.477	.477	.478	.479	.479	.480	.480	.481	.482
.62	.482	.483	.483	.484	.485	.485	.486	.487	.487	.488
.63	.488	.489	.490	.490	.491	.491	.492	.493	.493	.494
.64	.494	.495	.496	.496	.497	.498	.498	.499	.499	.500
.65	.501	.501	.502	.502	.503	.504	.504	.505	.505	.506
.66	.507	.507	.508	.508	.509	.510	.510	.511	.511	.512
.67	.513	.513	.514	.514	.515	.516	.516	.517	.517	.518
.68	.519	.519	.520	.520	.521	.522	.522	.523	.523	.524
.69	.525	.525	.526	.526	.527	.527	.528	.529	.529	.530
.70	.530	.531	.532	.532	.533	.533	.534	.535	.535	.536
.71	.536	.537	.537	.538	.539	.539	.540	.540	.541	.542
.72	.542	.543	.543	.544	.544	.545	.546	.546	.547	.547
.73	.548	.548	.549	.550	.550	.551	.551	.552	.553	.553
.74	.554	.554	.555	.555	.556	.557	.557	.558	.558	.559

Table **B2b** continued: **ln(1+*q*)**

q	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.75	.559	.560	.561	.561	.562	.562	.563	.563	.564	.565
.76	.565	.566	.566	.567	.567	.568	.568	.569	.570	.570
.77	.571	.571	.572	.572	.573	.574	.574	.575	.575	.576
.78	.576	.577	.577	.578	.579	.579	.580	.580	.581	.581
.79	.582	.583	.583	.584	.584	.585	.585	.586	.586	.587
.80	.588	.588	.589	.589	.590	.590	.591	.591	.592	.593
.81	.593	.594	.594	.595	.595	.596	.596	.597	.597	.598
.82	.599	.599	.600	.600	.601	.601	.602	.602	.603	.604
.83	.604	.605	.605	.606	.606	.607	.607	.608	.608	.609
.84	.610	.610	.611	.611	.612	.612	.613	.613	.614	.614
.85	.615	.615	.616	.617	.617	.618	.618	.619	.619	.620
.86	.620	.621	.621	.622	.622	.623	.624	.624	.625	.625
.87	.626	.626	.627	.627	.628	.628	.629	.629	.630	.630
.88	.631	.632	.632	.633	.633	.634	.634	.635	.635	.636
.89	.636	.637	.637	.638	.638	.639	,639	.640	.641	.641
.90	.642	.642	.643	.643	.644	.644	.645	.645	.646	.646
.91	.647	.647	.648	.648	.649	.649	.650	.650	.651	.652
.92	.652	.653	.653	.654	.654	.655	.655	.656	.656	.657
.93	.657	.658	.658	.659	.659	.660	.660	.661	.661	.662
.94	.662	.663	.663	.664	.664	.665	666	.666	.667	.667
.95	.668	.668	.669	.669	.670	.670	.671	.671	.672	.672
.96	.673	.673	.674	.674	.675	.675	.676	.676	.677	.677
.97	.678	.678	.679	.679	.680	.680	.681	.681	.682	.682
.98	.683	.683	.684	.684	.685	.685	.686	.686	.687	.687
.99	.688	.688	.689	.689	.690	.690	.691	.691	.692	.692

Table B3. Values of the NBD and LSD Parameters a (Adapted from Chattield [1969])

NBD: Values of $a = m/k$ for various values of c $= -m/\ln p_0 = -wb/\ln(1-b)$.
LSD : Values of $a = q/(1+q)$ for various values of w.

	Values of c for the NBD or w for the LSD									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.00	0.21	0.43	0.66	0.89	1.14	1.40	1.67	1.94	2.22
2	2.5 1	2.81	3.11	3.42	3.73	4.05	4.37	4.70	5.03	5.37
3	5.71	6.06	6.41	6.76	7.12	7.48	7.85	8.22	8.59	8.97
4	9.35	9.73	10.11	10.50	10.89	11.29	11.69	12.09	12.49	12.89
5	13.30	13.71	14.13	14.54	14.96	15.38	15.80	16.22	16.65	17.08
6	17.51	17.94	18.38	18.81	19.25	19.69	20.14	20.58	21.03	21.48
7	21.93	22.38	22.83	23.29	23.74	24.20	24.66	25.12	25.59	26.05
8	26.52	26.99	27.46	27.93	28.40	28.87	29.35	29.83	30.30	30.79
9	31.27	31.75	32.23	32.72	33.20	33.69	34.18	34.67	35.16	35.65
10	36.15	36.64	37.14	37.64	38.14	38.64	39.14	39.64	40.14	40.65
11	41.15	41.66	42.17	42.68	43.19	43.70	44.21	44.73	45.24	45.75
12	46.27	46.79	47.31	47.82	48.35	48.87	49.39	49.91	50.44	50.96
13	51.49	52.01	52.54	53.07	53.60	54.13	54.66	55.19	55.73	56.26
14	56.80	57.33	57.87	58.41	58.95	59.48	60.02	60.57	61.11	61.65
15	62.19	62.74	63.28	63.83	64.37	64.92	65.47	66.02	66.57	67.12
16	67.67	68.22	68.77	69.33	69.88	70.43	70.99	71.54	72.10	72.66
17	73.22	73.78	74.34	74.90	75.46	76.02	76.58	77.15	77.71	78.27
18	78.86	79.40	79.97	80.54	81.11	81.67	82.24	82.81	83.38	83.95
19	84.53	86.10	85.67	86.24	86.82	87.39	87.97	88.54	89.12	89.70

	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
20	90.3	96.1	101.9	107.9	113.8	119.9	125.9	132.1	138.2	144.4
30	150.6	156.9	163.2	169.6	176.0	182.4	188.9	195.4	201.9	208.4
40	215.0	221.6	228.3	234.9	241.6	248.4	255.1	261.9	268.7	275.5
50	282.3	289.2	296.1	303.0	309.9	316.9	323.9	330.9	337.9	344.9
60	352.0	359.1	366.2	373.3	380.4	387.6	394.7	401.9	409.1	416.3

Table B4. NBD and LSD Values of the Percentage of Buyers of an Item in one Period who Buy it again in Another Period of Equal Length

(For various values of b and w^*, as in Table 4.15)

$100\, b_R/b$	Average Purchase Frequency per Buyer, w							
	1.1	1.3	1.5	2	3	5	10	20
NBD, for Proportions of Buyers, b =								
.8					85	89	92	93
.6				70	78	84	88	90
.4		42	52	64	74	80	85	88
.2		37	47	60	70	77	83	86
.1	17	35	45	58	69	76	82	85
.05	16	34	45	58	68	76	82	85
LSD: Exact	16	34	44	57	68	75	81	85
Approximate	13	30	41	56	68	76	82	84

* Data for which $w < -\{\ln(1-b)\}/b$ cannot be fitted by an NBD.

Table B5. NBD and LSD Values of the Average Frequency of Purchase in a Period per Repeat-Buyer in an Equal Period

w_R	Average Purchase Frequency per Buyer, w							
	1.1	1.3	1.5	2	3	5	10	20
NBD, for Proportions of Buyers, b =								
.8					3.2	5.4	11	21
.6				2.2	3.4	5.7	11	22
.4		1.3	1.6	2.3	3.5	5.9	11	23
.2		1.4	1.7	2.4	3.7	6.0	12	23
.1	1.1	1.5	1.8	2.5	3.7	6.1	12	23
.05	1.2	1.5	1.8	2.5	3.8	6.1	12	23
LSD: Exact	**1.2**	1.5	1.8	2.5	3.8	6.2	12	23
Approximate	1.4	1.6	1.9	2.5	3.7	6.2	12	25

Table **B6.** NBD and LSD Values of the Percentage of Buyers of an Item in one Period who do NOT buy it in Another Equal Period

($100b_N/b = 100b_L/b = 100 - b_R/b$ from Table **B4**)

$100b_N/b$	Average Purchase Frequency per Buyer, w							
	1.1	1.3	1.5	2	3	5	10	20
NBD, for Proportions of Buyers, b =								
.8					15	11	8	7
.6				30	22	16	12	10
.4		58	48	36	26	20	15	12
.2		63	53	40	30	23	17	14
.1	83	65	55	42	31	24	18	15
.05	84	66	55	42	32	24	18	15
LSD: Exact	84	66	56	43	32	25	19	15
Approximate	87	70	59	44	32	24	18	16

Table B7. NBD and LSD Values of the Average Frequency of Purchase per "New" or "Lapsed" Buyer

($w_N = w_L$)

w_N	Average Purchase Frequency per Buyer, w							
	1.1	1.3	1.5	2	3	5	10	20
NBD, for Proportions of Buyers, b =								
.8					1.9	1.8	1.7	1.7
.6				1.6	1.6	1.6	1.6	1.6
.4		1.3	1.4	1.4	1.5	1.5	1.5	1.5
.2		1.2	1.3	1.4	1.4	1.5	1.5	1.5
.1	1.1	1.2	1.3	1.4	1.4	1.4	1.5	1.5
.05	1.1	1.2	1.3	1.4	1.4	1.4	1.4	1.4
LSD: Exact	1.1	1.2	1.3	1.3	1.4	1.4	1.4	1.4
Approximate	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4

Table B8. Penetration Growth

(NBD and LSD values of b_T/b_1 in time periods of relative lengths T, for different values of $100b_1$ and w_1 in a time-period of unit length, i.e. $T = 1$)

	T: Length of Time-Period as a Fraction of Base Period ($T = 1$)								
	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	4	12
b_T/b_1	$b_{\frac{1}{12}}/b_1$	$b_{\frac{1}{4}}/b_1$	$b_{\frac{1}{3}}/b_1$	$b_{\frac{1}{2}}/b_1$	$100b_1$	b_2/b_1	b_3/b_1	b_4/b_1	b_{12}/b_1
$100b_1 = 80$									
$w_1 = 20$	.58	.80	.85	.91	80	1.07	1.10	1.11	1.17
$= 10$	.45	.73	.80	.88	80	1.08	1.12	1.14	1.19
$= 5$	.31	.63	.71	.83	80	1.11	1.15	1.18	1.22
$= 3$	.22	.51	.62	.77	80	1.15	1.20	1.22	1.25
$100b_1 = 60$									
$w_1 = 20$	.53	.76	.81	.89	60	1.10	1.15	1.18	1.29
$= 10$	.43	.69	.76	.86	60	1.12	1.18	1.22	1.35
$= 5$	.30	.59	.68	.80	60	1.16	1.24	1.29	1.44
$= 3$	.21	.49	.59	.74	60	1.22	1.32	1.38	1.54
$= 2$	.15	.40	.50	.67	60	1.30	1.43	1.50	1.63
$100b_1 = 40$									
$w_1 = 20$	.51	.73	.79	.87	40	1.12	1.19	1.23	1.39
$= 10$	.41	.67	.74	.84	40	1.15	1.23	1.29	1.48
$= 8$	.29	.58	.66	.79	40	1.20	1.31	1.38	1.62
$= 3$	.21	.48	.58	.73	40	1.26	1.41	1.50	1.80
$= 2$	.15	.40	.50	.66	40	1.36	1.56	1.68	2.05
$= 1.5$	.12	.33	.43	.60	40	1.48	1.76	1.93	2.34
$= 1.3$	.11	.30	.40	.57	40	1.58	1.93	2.14	2.49

Table B8 continued

	T: Length of Time-Period as a Fraction of Base Period ($T = 1$)								
	$\frac{1}{12}$	1	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	4	12
b_T/b_1	$b_{\frac{1}{12}}/b_1$	$b_{\frac{1}{4}}/b_1$	$b_{\frac{1}{3}}/b_1$	$b_{\frac{1}{2}}/b_1$	$100b_1$	b_2/b_1	b_3/b_1	b_4/b_1	b_{12}/b_1
$100b_1 = 20$									
$w_1 = 20$	.49	.72	.77	.86	20	1.14	1.22	1.27	1.47
$= 10$	.39	.65	.72	.83	20	1.17	1.27	1.31	1.58
$= 5$	.29	.56	.65	.77	20	1.23	1.36	1.45	1.77
$= 3$	.21	.47	.57	.72	20	1.30	1.47	1.59	2.03
$= 2$	.15	.39	.49	.65	20	1.40	1.64	1.81	2.42
$= 1.5$	.12	.33	.43	.60	20	1.53	1.87	2.12	2.99
$= 1.3$	.11	.30	.39	.56	20	1.63	2.07	2.39	3.54
$100b_1 = 10$									
$w_1 = 20$	.48	.71	.77	.85	10	1.15	1.23	1.29	1.51
$= 10$	.39	.64	.72	.82	10	1.18	1.28	1.36	1.63
$= 5$	.28	.56	.64	.77	10	1.24	1.38	1.47	1.84
$= 3$	.21	.47	.56	.71	10	1.31	1.50	1.63	2.13
$= 2$	.15	.39	.49	.65	10	1.42	1.68	1.86	2.59
$= 1.5$	.12	.33	.42	.59	10	1.55	1.91	2.19	3.28
$= 1.3$	.11	.30	.39	.56	10	1.65	2.12	2.48	3.96
$= 1.1$	.09	.27	.35	.52	10	1.83	2.53	3.13	5.92

Table B8continued

	T: Length of Time-Period as a Fraction of Base Period (T = 1)								
	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	4	12
b_T/b_1	$b_{\frac{1}{12}}/b_1$	$b_{\frac{1}{4}}/b_1$	$b_{\frac{1}{3}}/b_1$	$b_{\frac{1}{2}}/b_1$	$100b_1$	b_2/b_1	b_3/b_1	b_4/b_1	b_{12}/b_1
$100b_1 = 5$									
w_1 = 20	.48	.70	.76	.85	5	1.15	1.24	1.30	1.53
= 10	.39	.64	.71	.82	5	1.18	1.29	1.37	1.66
= 5	.28	.55	.64	.77	5	1.24	1.39	1.49	1.88
= 3	.21	.47	.56	.71	5	1.32	1.51	1.65	2.18
= 2	.15	.39	.49	.65	5	1.42	1.69	1.89	2.66
= 1.5	.12	.33	.42	.59	5	1.55	1.93	2.22	3.40
= 1.3	.11	.30	.39	.56	5	1.66	2.14	2.52	4.15
= 1.1	.09	.27	.35	.52	5	1.84	2.55	3.17	6.31
$100b_1$ =0 (LSD)									
w_1 = 20	.47	.70	.76	.85	0	1.15	1.24	1.31	1.55
= 10	.38	.64	.71	.82	0	1.19	1.30	1.38	1.68
= 5	.28	.55	.64	.76	0	1.25	1.40	1.50	1.91
= 3	.20	.47	.56	.71	0	1.32	1.52	1.67	2.23
= 2	.15	.39	.48	.65	0	1.43	1.71	1.91	2.74
= 1.5	.12	.33	.42	.59	0	1.56	1.95	2.25	3.53
= 1.3	.11	.30	.39	.56	0	1.66	2.16	2.55	4.33
= 1.1	.09	.27	.35	.52	0	1.84	2.57	3.21	6.63

Table B9. The Average Purchase Frequency in Periods of Different Lengths

(NBD and LSD values of w_T/w_1 in time-periods of relative lengths T, for different values of $100b_1$ and w_1 in a time-period of "unit" length, i.e. $T = 1$)

	T: Length of Time Period as a Fraction of Base Period ($T = 1$)								
	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	4	12
w_T/w_1	$w_{\frac{1}{12}}/w_1$	$w_{\frac{1}{4}}/w_1$	$w_{\frac{1}{3}}/w_1$	$w_{\frac{1}{2}}/w_1$	w_1	w_2/w_1	w_3/w_1	w_4/w_1	w_{12}/w_1
$100b_1 = 80$									
	.14	.31	.39	.55	20	1.88	2.74	3.59	10.29
	.18	.34	.42	.57	10	1.85	2.68	3.51	10.06
	.27	.40	.47	.60	**5**	1.80	2.60	3.40	9.80
	.38	.49	.54	.65	3	1.74	2.51	3.29	9.63
$100b_1 = 60$									
	.16	.33	.41	.56	20	1.82	2.61	3.39	9.31
	.20	.36	.44	.58	10	1.78	2.54	3.27	8.91
	.28	.42	.49	.62	**5**	1.72	2.41	3.10	8.36
	.39	.51	.56	.67	3	1.64	2.27	2.89	7.82
	.54	.62	.66	.74	2	1.54	2.09	2.66	7.35
$100b_1 = 40$									
	.16	.34	.42	.57	20	1.79	2.53	3.25	8.64
	.20	.37	.45	.60	10	1.74	2.44	3.11	8.13
	.29	.43	.50	.63	**5**	1.67	2.30	2.90	7.42
	.40	.52	.58	.69	3	1.58	2.13	2.67	6.67
	.54	.63	.67	.76	2	1.47	1.93	2.38	5.85
	.69	.75	.78	.83	1.5	1.35	1.71	2.07	5.13
	.79	.82	.84	.88	1.3	1.26	1.55	1.87	4.83

Table B9 continued

	T: Length of Time Period as a Fraction of Base Period (T = 1)								
	$\frac{1}{12}$	$\frac{1}{4}$	1	$\frac{1}{2}$	1	2	3	4	12
w_T/w_1	$w_{\frac{1}{12}}/w_1$	$w_{\frac{1}{4}}/w_1$	$w_{\frac{1}{3}}/w_1$	$w_{\frac{1}{2}}/w_1$	w_1	w_2/w_1	w_3/w_1	w_4/w_1	w_{12}/w_1
$100b_1 = 20$									
	.17	.35	.43	.58	20	1.76	2.47	3.15	8.14
	.21	.38	.46	.61	10	1.71	2.37	2.99	7.57
	.29	.45	.51	.65	5	1.63	2.21	2.77	6.77
	.40	.53	.59	.70	3	1.54	2.04	2.51	5.92
	.55	.64	.68	.76	2	1.43	1.83	2.21	4.96
	.70	.75	.78	.84	1.5	1.31	1.60	1.89	4.01
	.79	.83	.85	.89	1.3	1.23	1.45	1.67	3.39
$100b_1 = 10$									
	.17	.35	.43	.59	20	1.75	2.44	3.10	7.93
	.21	.39	.46	.61	10	1.70	2.34	2.95	7.34
	.29	.45	.52	.65	5	1.62	2.18	2.71	6.51
	.41	.53	.59	.70	3	1.53	2.00	2.45	5.63
	.55	.64	.68	.77	2	1.41	1.79	2.15	4.64
	.70	.76	.78	.84	1.5	1.29	1.57	1.83	3.86
	.79	.83	.85	.89	1.3	1.21	1.42	1.61	3.03
	.92	.93	.94	.95	1.1	1.09	1.18	1.28	2.03

Table B9 continued

	T: Length of Time Period as a Fraction of Base Period ($T = 1$)								
	1	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	4	12
w_T/w_1	$w_{\frac{1}{12}}/w_1$	$w_{\frac{1}{4}}/w_1$	$w_{\frac{1}{3}}/w_1$	$w_{\frac{1}{2}}/w_1$	w_1	w_2/w_1	w_3/w_1	w_4/w_1	w_{12}/w_1
$100b_1 = 5$									
	.17	.36	.44	.59	20	1.74	2.43	3.08	7.84
	.22	.39	.47	.61	10	1.69	2.32	2.92	7.24
	.30	.45	.52	.65	5	1.61	2.16	2.69	6.39
	.41	.53	.59	.70	3	1.52	1.99	2.43	5.50
	.55	.64	.69	.77	2	1.41	1.77	2.12	4.51
	.70	.76	.79	.84	1.5	1.29	1.55	1.80	3.52
	.79	.83	.85	.89	1.3	1.21	1.40	1.59	2.89
	.92	.93	.94	.95	1.1	1.09	1.18	1.26	1.90
$100b_1 = 0$ (LSD)									
	.18	.36	.44	.59	20	1.74	2.42	3.06	7.75
	.22	.39	.47	.61	10	1.68	2.31	2.90	7.14
	.30	.45	.52	.65	5	1.60	2.15	2.67	6.29
	.41	.54	.60	.71	3	1.51	1.97	2.40	5.39
	.55	.64	.69	.77	2	1.40	1.76	2.09	4.38
	.70	.76	.79	.84	1.5	1.28	1.54	1.78	3.40
	.79	.83	.85	.89	1.3	1.20	1.39	1.57	2.77
	.92	.93	.94	.96	1.1	1.09	1.17	1.25	1.81